



Western Australian Certificate of Education ATAR course examination, 2016

Question/Answer booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

Student number: In figures

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In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	12	12	100	97	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed**65% (97 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

Working time: 100 minutes.

Question 9**(5 marks)**

Consider the integral $I = \int x\sqrt{(1+x)^n} dx$, where n is any positive integer.

Using the substitution $u = 1 + x$ and an appropriate anti-derivative, develop a simplified expression for I in terms of x and n .

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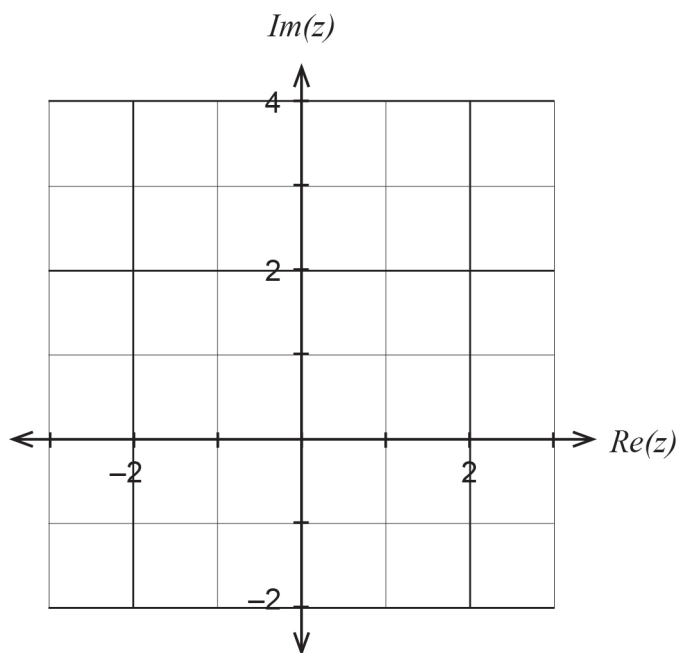
Question 10

(9 marks)

On the Argand planes below sketch the locus of the complex number $z = x + iy$ given by:

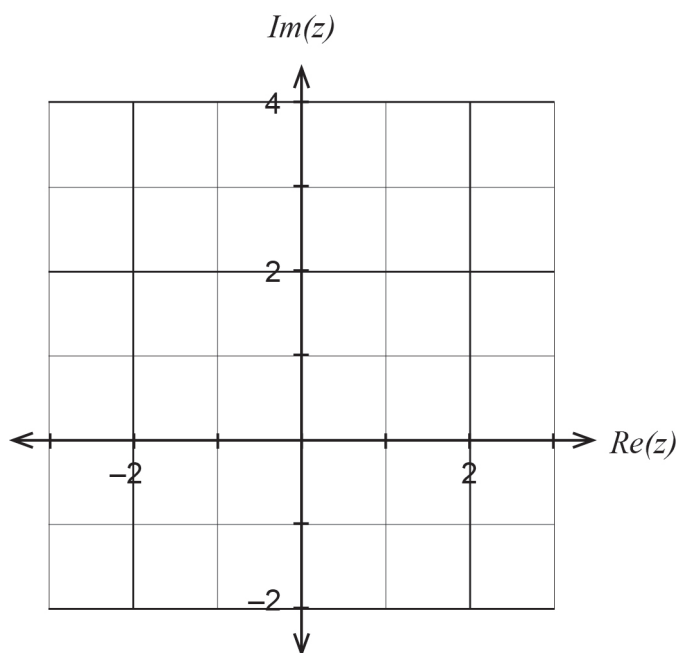
(a) $|z - 2i| = 1$.

(3 marks)



(b) $|z - 1 + i| \geq |z + 1 - i|$.

(3 marks)



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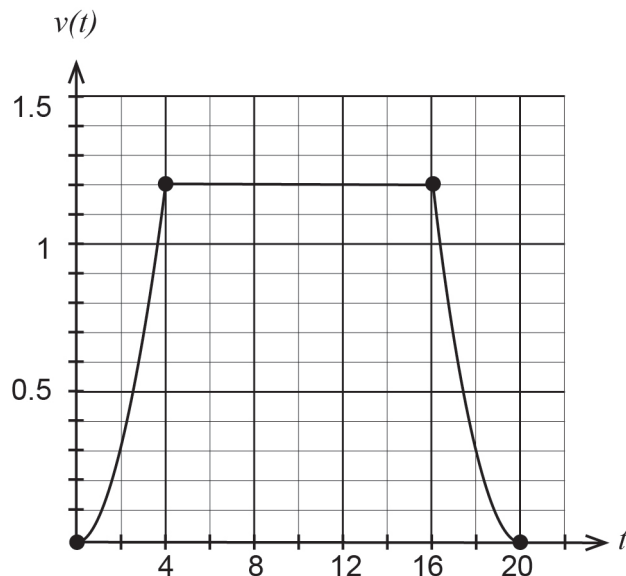
- (c) For the locus $|z - 2i| = 1$ from part (a), state the exact maximum value for $|z + 2|$.
(3 marks)

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Question 11

(7 marks)

A lift goes up within a high rise building so that its velocity $v(t)$ is given by the graph shown below. The maximum velocity of the lift during its ascent is 1.2 ms^{-1} . For the first four seconds, the acceleration is given by $a(t) = kt$. For the final four seconds of its ascent, the lift decelerates at the same rate.



- (a) Show that the value of the constant $k = \frac{3}{20}$. (2 marks)

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- (b) Using the incremental formula, determine the approximate change in velocity v from $t = 2$ to $t = 2.1$ seconds. (2 marks)

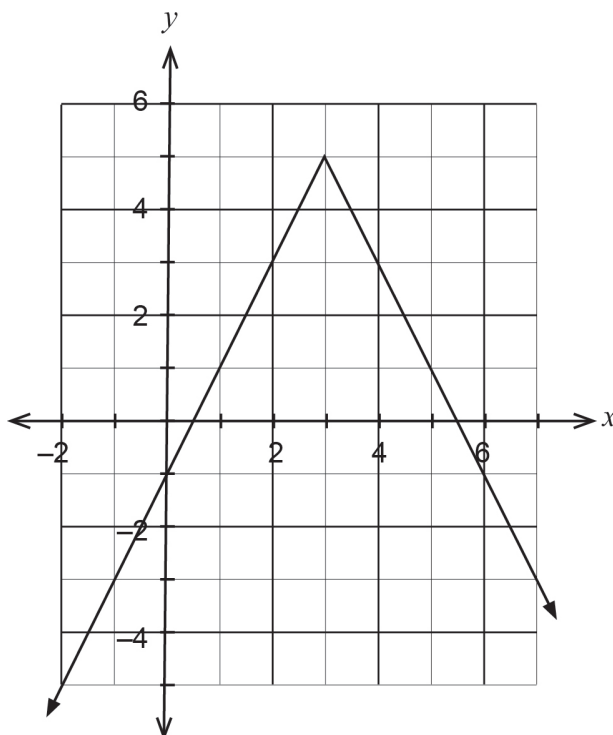
- (c) Determine the total distance that the lift travels upwards during its ascent, correct to the nearest 0.1 m. (3 marks)

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Question 12

(6 marks)

The graph of $f(x) = a|x - b| + c$ is shown below.



- (a) Determine the values for the constants a , b and c .

(3 marks)

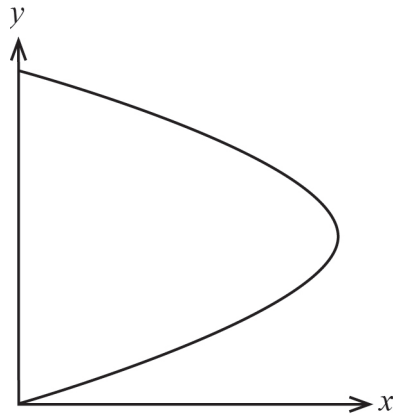
Consider the equation $|f(x)| = d$.

- (b) If the equation $|f(x)| = d$ has exactly four solutions, state the possible value(s) for the constant d . Explain. (3 marks)

Question 13

(5 marks)

The graph of the curve $2x = \sin(y)$ is sketched for $0 \leq y \leq \pi$.



- (a) Determine the expression for $\frac{dy}{dx}$ in terms of y . (2 marks)

- (b) Determine the area of the region bounded by the curve $2x = \sin(y)$ and the y axis. (3 marks)

Question 14**(7 marks)**

Consider the complex equation $z^4 = -16i$.

- (a) Solve the equation giving all solutions in the form $r \operatorname{cis} \theta$ where $-\pi < \theta \leq \pi$. (4 marks)

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Let w be the solution to $z^4 = -16i$ that has the least positive argument.

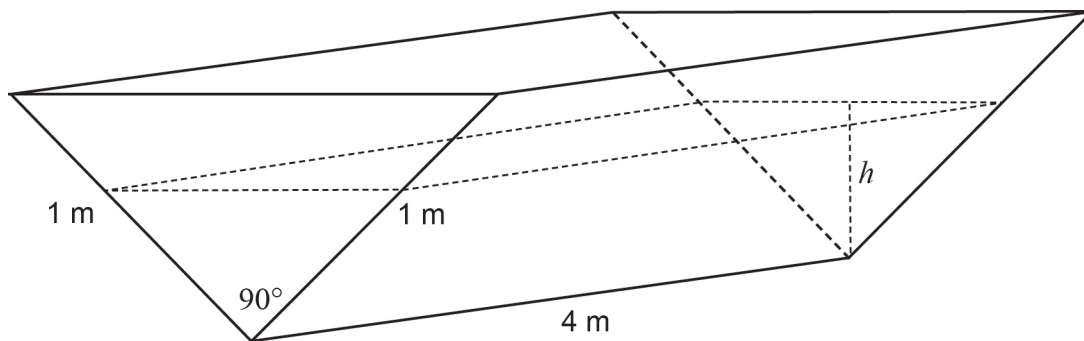
- (b) Determine the value for $\arg(w + 2)$. (3 marks)

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Question 15

(10 marks)

A four metre long water tank, open at the top, is in the shape of a triangular prism. The triangular face is a right isosceles triangle with congruent sides of one metre length.



Initially the tank is completely full with water, but it develops a leak and loses water at a constant rate of 0.08 cubic metres per hour.

Let h = the depth of water, in metres, in the tank after t hours.

- (a) Show that the volume of water in the tank V cubic metres, is given by the expression $V(h) = 4h^2$. (2 marks)

- (b) Determine the rate of change of the depth, correct to the nearest 0.01 metres per hour, when the depth is 0.6 metres. (3 marks)

Assume that the rate of leakage stays constant at 0.08 cubic metres per hour.

- (c) Show that the differential equation that relates $\frac{dh}{dt}$ with the depth h is given by $\frac{dh}{dt} = -\frac{1}{100h}$. (1 mark)

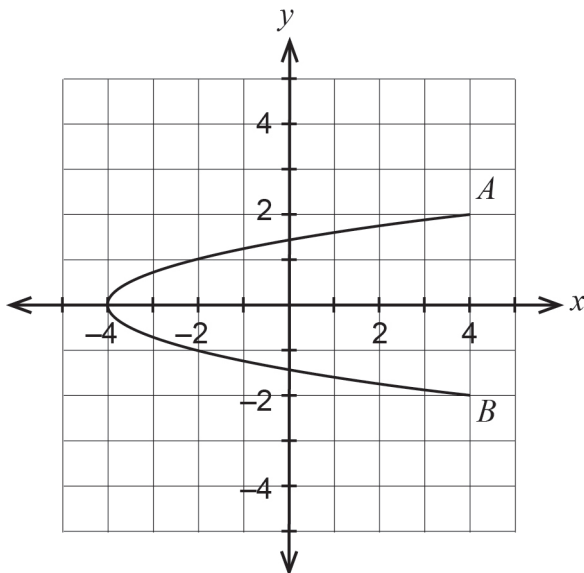
- (d) Hence determine the relationship for the depth h at any time t hours. (4 marks)

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Question 16

(10 marks)

A particle's position vector $\underline{r}(t)$ is given by $\underline{r}(t) = \begin{pmatrix} 4 \cos 2t \\ 2 \cos t \end{pmatrix}$ centimetres where t is measured in seconds. A plot of the path of the particle is shown below.



(a) Express the path of the particle as a Cartesian equation.

(3 marks)

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- (b) Determine the speed of the particle, correct to 0.01 cm per second, when it first reaches the point where $x = -2$. (4 marks)

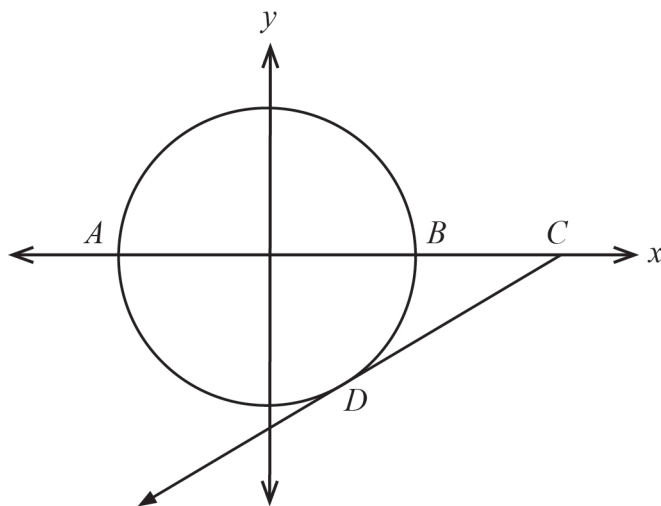
- (c) Write the expression, in terms of trigonometric functions, for the distance the particle will travel along its path in travelling from point A to point B . Do **not** evaluate this expression. (3 marks)

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Question 17

(8 marks)

The diagram shows a circle with equation $x^2 + y^2 = 16$ with points A, B being the horizontal intercepts of this circle. DC is the tangent to the circle at point D , intersecting the x axis at point C . Point D has coordinates $(2, -2\sqrt{3})$.



- (a) Show that the equation for the tangent at point D can be written in the form $\sqrt{3}y = x - 8$.

(3 marks)

- (b) Determine the coordinates of point C . (1 mark)

The region bounded by the arc AD , the line segment \overline{DC} and the x axis is rotated about the x axis.

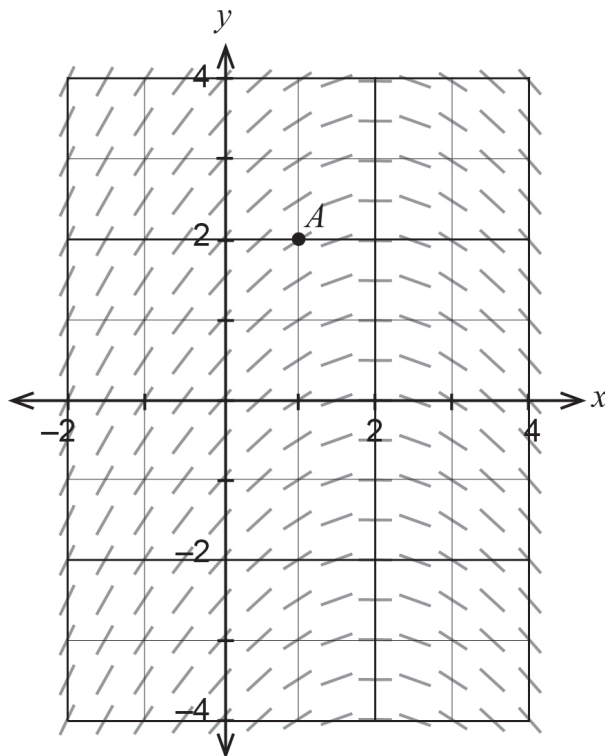
- (c) Determine the volume of the resulting solid, correct to the nearest 0.01 cubic units. (4 marks)

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Question 18

(7 marks)

A first-order differential equation has a slope field as shown in the diagram below.



- (a) Determine the general differential equation that would yield this slope field. (3 marks)

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The slope field at point $A(1, 2)$ has a value of 0.5.

- (b) Determine the equation for the curve $y = f(x)$ containing point A . (4 marks)

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Question 19

(16 marks)

The volume of water used by the SavaDaWater company to top up an ornamental pool has been observed to be normally distributed with mean $\mu = 175$ litres and standard deviation $\sigma = 15$ litres.

The ornamental pool is topped up 50 times. Determine the probability that the:

(a) sample mean volume will be between 173 and 177 litres. (3 marks)

(b) total volume of water used is less than 8.96 kilolitres. (3 marks)

Water is a scarce commodity and accuracy is required. The pool is topped up 50 times and the sample mean obtained is denoted by \bar{W} .

(c) If it is required that $P(a \leq \bar{W} \leq b) = 0.99$, then determine the values of a and b , each correct to 0.1 litres. (3 marks)

- (d) If the probability for the mean amount of water used differs from μ by less than five litres is 96%, find n , the number of waterings that need to be measured. (3 marks)

A rival company called WollliWorks takes over the watering of the ornamental pool. Over 36 consecutive days, it was observed that the WollliWorks company used a total of 6.57 kilolitres. The standard deviation for the 36 days was also 15 litres.

A representative from the SavaDaWater company states that 'WollliWorks are using significantly more water than we did when we were filling this pool. They are wasting water'.

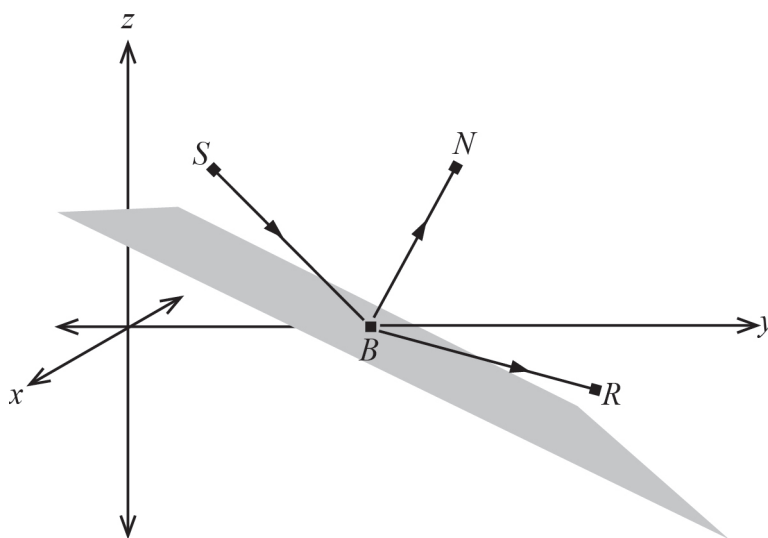
- (e) Perform the calculations necessary to comment on this claim. (4 marks)

Question 20

(7 marks)

A laser pointer at point S directs a highly focused beam of light towards a mirror. The beam bounces off the mirror at point B and is then reflected away from the mirror toward point R .

The mirror's surface is given by the equation $\vec{r} \cdot (\vec{j} + 2\vec{k}) = 9$ and the laser pointer is positioned at point S with position vector $-2\vec{i} + 3\vec{j} + 6\vec{k}$. The laser pointer is held so that the beam is pointed in the direction $\vec{d}_1 = \vec{i} + \vec{j} - \vec{k}$.



- (a) Determine the position vector for point B .

(4 marks)

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The laser beam is reflected away from the mirror so that:

- the angle of the incoming beam \vec{SB} to the normal of the mirror is equal to the angle of the reflected beam \vec{BR} to the normal of the mirror i.e. $s\angle SBN = s\angle NBR$.
- the incoming beam \vec{SB} , the normal of the mirror and the reflected beam \vec{BR} are all contained in one plane.

Let \hat{d}_2 = the unit vector in the direction of the reflected beam \vec{BR} i.e. $|\hat{d}_2| = 1$.

- (b) Determine the unit vector \hat{d}_2 giving components correct to 0.01. (3 marks)

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Additional working space

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